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Essay and Opinion

Cosmological Black Holes
Version 1

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Abstract We present in this essay the cosmological black holes where unlike standard black holes seeing asymptotically Minkowski flat spacetime, the black hole is embedded into a cosmological spacetime. We put into historical context the birth of this idea and describe the physical consequences and astrophysical applications of such a black hole.

The very concept of black hole (BH) comes back to dark ages, way before general relativity (GR). Indeed the idea that light could not escape from an accumulation of matter was proposed in 1783 by the reverend John Michell [?] sending his work to his friend Cavendish for him to present it to the Royal Society. From Rømer's studies of the eclipses of Jupiter's moon Io [?], a determination of the speed of light was possible. Assuming light has a corpuscular nature, some believed it could be affected by gravity just as the rest of the matter.

In 1796, Pierre Laplace, in the troubled times of the french revolution, presumably unaware of Michell's work, postulated the same type of object: a dark star so compact that light could not escape [?]. On a request of von Zach, he provided a mathematical proof of his idea in 1799 [?]: he worked out the amount of mass M present in such a star of radius r_s and found the same result as Michell:

$$r_s = \frac{2GM}{c^2}, \tag{1}$$

where c is the speed of light and G the gravitational constant. Interestingly, this value is in agreement with the GR result which was only worked out later as Einstein introduced his equations in 1915 [? ?]:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

The same year of the birth of GR, Karl Schwarzschild [?], between two assaults of the german army, discovered a remarkable solution to equation (??): the Schwarzschild solution which line element reads:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{3}$$

from now $c = 1$. This solution describes gravity processes in the exterior of a spherically symmetric star. At $r = r_s$, the Schwarzschild radius, a (coordinate) singularity appears. However at that time, it was believed stars with a radius $r < r_s$ were pathological and not present in Nature. Observe also that this solution is asymptotically flat as the metric is Minkowski for $r \to \infty$.

A second groundbreaking discovery was the application of Einstein equations (??) to cosmological purposes leading in the 20s to the introduction of expanding spacetimes by Friedmann and Lemaître [? ?] (FLRW spacetime thereafter) and eventually the observational confirmation that our universe was expanding by Hubble [?] helped by Humason in 1927. Soon after, models of collapsing structure in an otherwise expanding spacetime were proposed. The simplest and most famous model might be the Einstein-Strauss model [?] which is nothing but a Schwarzschild spacetime glued to a FLRW universe. However as these objects are time symmetric, they cannot describe a dynamical accretion within our universe. In 1933, the McVittie solution of (??) was exhibited as a try to describe a collapsing structure into an otherwise expanding spacetime [?]. It corresponds to a FLRW spacetime as $r \to \infty$ and to (??) near the horizon, however it has been discarded as a viable solution, one of the reason being that the geometrical sector of the theory was overconstraining the matter sector.

In 1939, a very important paper for black hole physics was released by Oppenheimer and Snyder. They proposed for the first time a study of a spherically symmetric gravitational collapse within GR [?]. They showed that the collapse would occur during a finite time and stop at the Schwarzschild radius thus leading to the formation of a black hole! Some time was required for the scientific community to digest this result and together with the advent of the computing power (one can mention the work of the group of Colgate [?] and the soviet equivalent by the group of Zel'dovich [?]) in the 60s, the concept and word black hole were well accepted among scientist at the image of Wheeler who started as a skeptic to this new astrophysical object but then turned around and even christened the new object "black hole" [?]. From those difficult starts, the golden age for BH physics could start with a plethora of new discoveries: Eddington-Finkelstein and Kruskal coordinate systems [? ?], Kerr solution [?], the no-hair theorem [?], the singularity theorems [?], Hawking radiation [?].

Modern theoretical research on black hole includes: the exploration of the quantum properties of BH [?], of new types of BH in higher dimension [?] or with new matter field or with new way to described gravitational processes (modified gravity) [?]. In this essay, we stress how changing the assumption of asymptotic flatness discussed in equation (??) would impact on the BH dynamics and formation. We will call these objects cosmological black holes (CBH) to be contrasted with astrophysical black holes (ABH) which are static or asymptotically stationary.

So far, we have seen the naive and historical tries to model a collapsing structure in an otherwise expanding background. In this essay, we focus on a model involving the Lemaître-Tolman-Bondi (LTB) metric [? ?]. Other modern attempts include [? ? ? ? ? ? ? ?]. The LTB metric is:

$$ds^2 = dt^2 - \frac{\left[\partial_r R(t,r)\right]^2}{1 + f(r)} dr^2 - R(t,r)^2 d\Omega^2. \tag{4}$$

$f(r)$ is the curvature of the spacetime and $R(t,r)$ encodes all the dynamics of the spacetime. Using the Einstein equations (**??**) gives the equation of motion for $R(t,r)$:

$$\left(\frac{\partial R(t,r)}{\partial t}\right)^2 = f(r) + \frac{2M(r)}{R(t,r)}, \tag{5}$$

where $M(r)$ is the Misner-Sharp mass of the system [**?**] and is defined as an integration constant: $M'(r) \equiv \frac{1}{2}\rho(t,r)R'(t,r)R(t,r)$. Depending on the sign of the curvature term $f(r)$, it is possible to find solutions for (**??**). In order to model CBH, we present the following special case:

$$f(r) = -re^{-r}, \tag{6}$$

$$M(r) = \frac{r^{3/2}}{a}(1 + r^{3/2}), \tag{7}$$

where a is a constant with which depends on the initial conditions of the collapse. With this toy model, (**??**) can be solved for $R(t,r)$ and the solution has been shown to describe a collapsing structure within an expanding universe [**?**]. Observe that this solution describes an overdensity as $f(r) < 0$ but corresponds to an asymptotically flat universe: $\lim_{r\to\infty} f(r) = 0$. We stress also that $\lim_{r\to\infty} M(r) \propto r^3$ corresponds to a FLRW space-time.

The main difference between CBH and ABH is that most of the standard concepts of BH have to be redefined: the horizon of CBH becomes dynamical leading to: Hayward's trapping horizon [**?**], isolated horizon [**?**], Ashtekar and Krishnan's dynamical horizon [**?**], and Booth and Fairhurst's slowly evolving horizon [**?**]. Some relations between those horizons within the different models remains to be explored. Moreover, not only the horizon of the BH but also the mass in these setups is different [**?**].

While its has been shown that, in a collapse, the horizon of ABH can evolve, the uniqueness theorems stipulate that the asymptotic state of a neutral collapsing structure is a Kerr BH [**?**]. Therefore, one can ask whether the CBH presented here have any astrophysical consequence. A partial answer has been given in [**?**] where the lensing properties of the CBH were computed. The key point is that the compacter the BH, the closer from the ABH case the lensing is. With the current data on the deviation angle and the time delay, no difference between CBH and ABH was reported.

Hawking radiation which is one of most interesting properties of a BH has been revisited in the CBH case. The first obvious difference is that the created particles will not see the Minskowski vaccum a Bunch-Davies vaccum for the FLRW background [**?**]. Paradoxically while the rest of the field is still at its infancy, many works exist for particle production in CBH scenarios. More precisely, a whole community is interested in Hawking radiation for (anti-)de Sitter-Schwarzchild BH, with topic ranging from semiclassical techniques for scalar particle production [**?**] to group definition of vaccum for gravitational waves [**?**]. In the case of the class of CBH presented in this essay, the nature of the dynamical horizon changes the thermality [**?**] and the radiation spectrum [**?**].

CBH could be also insightful for early universe cosmology. Primordial black holes are small BH which formed in the very early universe and either are used to experimentally test the radiation properties of BH or are used as seed for super massive black holes. Constraints were put on the maximal mass of these black holes in order not to spoil big-bang Nucleosynthesis, for instance $M < 10^5 M_\odot$ in [**?**] at the neutron-proton freeze out. Similar constraints are expected for the CBH case [**?**].

We argued here that the CBH is a fertile and necessary way to model BH. Not only the models discussed here provide novel features of black hole but they fit well in the global landscape of modern BH theory. The quantum and semiclassical properties of CBH, as well as of ABH still contains many secrets. Many classical theorems about BH listed in this essay do not have yet a counter-part in the CBH case. We suggest that CBH could be relevant to understand the production of gamma ray bursts, for instance within the fireshell model [**?**]. CBH models could also be applied to more diverse astrophysical scenarios: shadow of BH, accretion disk modeling and X-ray emission, and, last but not least, to apply those ideas to investigate a black hole merger is a challenge to be tackled.